

# ON THE CAPACITY REGION FOR COGNITIVE MULTIPLE ACCESS OVER WHITE SPACE CHANNELS

Huazi Zhang, Huaiyu Dai<sup>†</sup> and Zhaoyang Zhang<sup>‡</sup>

## Abstract

Cognitive Radio (CR) has stirred great interest with its potential to exploit the already scarce spectrum resource. The cognitive multiple access channels (CogMAC) are commonly seen in many applications, such as the Cognitive Radio Sensor Networks (CRSN), and the 802.22 cognitive Wireless Regional Area Networks (WRAN). In this paper, the primary user (PU) activities are treated as on/off side information, which can be obtained causally or non-causally. The cognitive MAC channels are then modeled as multi-switch channels and their rate regions are characterized. Explicit forms of outer and inner bounds of this rate region are derived by assuming additional side information, and they are shown to be tight in some special cases. An optimal rate and power allocation scheme that maximizes the sum rate is also proposed. The numerical results show the importance of side information in enhancing the capacity and the effectiveness of our rate allocation scheme.

## Index Terms

Huazi Zhang (e-mail: thomas25@163.com) is with the Department of Information Science and Electronic Engineering, Zhejiang University, and is currently a visiting Ph.D. student in North Carolina State University, under the joint supervision of Huaiyu Dai and Zhaoyang Zhang. Huaiyu Dai (e-mail: huaiyu\_dai@ncsu.edu) is with the Department of Electrical and Computer Engineering, North Carolina State University, USA. Zhaoyang Zhang (e-mail: ning\_ming@zju.edu.cn) is with the Department of Information Science and Electronic Engineering, Zhejiang University, China.

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Cognitive MAC channel, Capacity of Cognitive Radio Networks, Optimal Rate allocation,  
Three-switch Cognitive Channel, Cognitive Radio Sensor Network, 802.22 WRAN.

## I. INTRODUCTION

### A. Motivation

Wireless data traffic is expected to continuously grow in the foreseeable future, and the fixed spectrum allocation policy can no longer meet the exponentially increasing bandwidth demand. However, the wireless resource is actually under-utilized in terms of spectrum occupancy in many scenarios [1]. Cognitive Radio (CR) technology allows the unlicensed cognitive users, or Secondary Users (SU), to share the spectrum resources with the licensed Primary Users (PU), and receives great interest recently.

Multiple access (MAC) communications are common in wireless systems, and cognitive radio (CR) networks are no exceptions. In particular, cognitive MAC can be found in the Cognitive Radio Sensor Networks (CRSN) [2] and 802.22 cognitive Wireless Regional Area Networks (WRAN) [3]. In both networks, SUs are partitioned into clusters according to their locations and common channels [4]. The cluster members (CM) send the sensed information to cluster head (CH) through a cognitive MAC in the uplink.

Therefore, the results on the rate region of cognitive MAC channel allow us to estimate the performance of CRSN and 802.22 systems. Intuitively, both of their rate regions are inevitably influenced the PU activities. Equipped with intrinsic spectrum sensing capability, SUs can obtain the states of PU activities, either causally or non-causally. Thus, PU activities can be viewed as side information at both cognitive transmitters and receivers, and information theoretic results on channel capacity with side information can be adopted to analyze their rate regions.

### B. Related Works

The existing studies on the capacity of cognitive channels are mainly conducted in the context of interference channels [5], further divided into the underlay [6] and overlay [7], [8] cases. The former assumes that the secondary user (SU) has the channel knowledge and can control its transmission power to restrict the interference to the primary users (PUs). In contrast, the overlay approach models the coexistent communications as the “interference channel with degraded message sets (IC-DMS)” [9], [10], and employs intricate coding schemes for capacity enhancement.

The researches on cognitive network capacity are fueled by the seminal work of Kumar [11]. For cognitive networks, the primary and secondary network can achieve scaling as two

standalone network [12] [13]. Recently, with the help of highway system [14], the throughput of both networks is further improved [15], which is the best known till now.

In the above study, concurrent transmissions of primary and secondary systems is allowed. To effectively control the interference, either channel gains of the SU-PU links or PU messages are assumed known at the secondary transmitter. However, many practical cognitive systems adopt a “sense and access” paradigm, in which SUs first identify the states of the PU occupation through spectrum sensing, and only access the *white space channels*<sup>1</sup> to avoid undesired interference. Thus, locally sensed PU activities can be viewed as side information about the channel state at both cognitive transmitters and receivers, and information theoretic results on channel capacity with side information [16] can be adopted to analyze their rate regions.

### C. Summary of Contributions

To the best of our knowledge, the pioneering work [17] is the first to consider the “sense and access” diagram, in which the PU activities sensed at the cognitive transmitter and receiver is modeled as on/off side information, and the capacity of a two-switch cognitive channel is explored. Motivated by this work, in this paper we extend the study to a cognitive multiple access scenario. The contributions of this paper are summarized below:

#### 1) Achievable Rate Region of the Cognitive MAC Channel:

By viewing the primary user activities around the transmitters and receiver as side information, we model the memoryless cognitive MAC channel as a three-switch MAC channel. The achievable regions of the three-switch MAC channel are derived for two scenarios with independent causal and non-causal side information at the transmitters, and a special case in which the receiver has strong spectrum sensing capacity.

#### 2) Outer and Inner Bounds:

The capacity regions of the three-switch MAC channel derived are intractable in general. We further obtain explicit outer and inner bounds of the capacity region by assuming additional side information at the transmitters or receiver. It is found that the outer and inner bounds coincide in two special cases, when the side information between the

<sup>1</sup>The *white space channel* is the spectrum resource in frequency domain that is temporarily unoccupied by primary users.

transmitters and receiver are highly correlated, or when the states of PU signals change slowly.

- 3) **Sum-rate Optimal Rate/Power Allocation:** We optimize the rate and power allocation between transmitters when both the transmitters and receiver have global side information, and analyze the impact of two parameters, PU occupation probability and correlation in side information, on the sum rate.
- 4) **Extension to the fading scenario and general  $m$ -user cases:** We further analyze a general model with fading and interference, in which the receiver is active all the times. Furthermore, the results for general  $m$ -user case are also obtained.

The rest of the paper is organized as follows. In Section II, we introduce the memoryless cognitive MAC channel and model it as a three-switch MAC channel. The main results of rate expressions, bounds and rate/power allocation are listed in Section III, with relevant derivations and analysis given in Section IV. Numerical results are presented in Section V, and the whole paper is concluded in Section VI.

## II. PROBLEM FORMULATION AND SYSTEM MODEL

### A. Three-switch Cognitive MAC Channel

The cognitive MAC channel with neighboring primary activities is illustrated in Fig. 1. We follow [17] and model the memoryless cognitive MAC channel as a three-switch equivalent channel, as in Fig. 1. The states of switches at the two cognitive transmitters CT1, CT2 and the cognitive receiver CH are denoted respectively as  $S_{T_1}$ ,  $S_{T_2}$  and  $S_R$ , taking values of either 1(on) or 0(off). When the cognitive users detect (strong enough) interference from PU signals, the switch is turned off to avoid collisions. Otherwise, the switch is turned on for opportunistic communications. In this work, we assume perfect spectrum sensing for ease of discussion. Based on this model, the input and output of the channel is related as:

$$Y = (S_{T_1}X_1 + S_{T_2}X_2 + Z) S_R, \quad (1)$$

$$S_{T_1}, S_{T_2}, S_R \in \{0, 1\},$$

where  $X_1$  and  $X_2$  are the transmitted symbols of CT1 and CT2 with average power constraint  $E[|X_i|^2 S_{T_i}] \leq P_i, i = 1, 2$ , and  $Z$  is the AWGN noise with unit variance.

### III. MAIN RESULTS

#### A. Achievable Rate and Capacity Regions of Cognitive MAC Channel

We first explore the rate regions of the memoryless cognitive MAC channel with independent transmitter side information, either causal or non-causal. Note that the causality of the receiver side information does not matter in the study, as the receiver can decode after the transmission is finished. In the special case that the transmitter side information is also known at the receiver, we can derive the capacity region. These results are natural extension of those in [17] concerning the capacity of a single-link two-switch channel.

##### 1) Causal Side Information at the Transmitters.

*Theorem 1:* For the three-switch MAC channel with independent<sup>2</sup> *causal* side information  $S_{T_1}$  and  $S_{T_2}$  at the transmitters and side information  $S_R$  at the receiver, coding can be performed directly on the input alphabets (i.e.,  $U_1 = X_1$ ,  $U_2 = X_2$ ) and an achievable rate region is given by:

$$\mathcal{R}_{S_{T_1}, S_{T_2}, S_R}^{\text{causal}} \triangleq \bigcup \left\{ (R_1, R_2) : \begin{array}{l} R_1 \leq \max_{p(X_1)p(X_2)} I(X_1; Y, S_R | X_2) \\ R_2 \leq \max_{p(X_1)p(X_2)} I(X_2; Y, S_R | X_1) \\ R_1 + R_2 \leq \max_{p(X_1)p(X_2)} I(X_1, X_2; Y, S_R) \end{array} \right\}, \quad (2)$$

where  $\bigcup$  denotes the convex hull of all rate pairs.

##### 2) Non-causal Side Information at the Transmitters.

*Theorem 2:* For the three-switch MAC channel with independent *non-causal* side information  $S_{T_1}$  and  $S_{T_2}$  at the transmitters and side information  $S_R$  at the receiver, coding can be performed directly on the input alphabets (i.e.,  $U_1 = X_1$ ,  $U_2 = X_2$ ) and an achievable rate region is given by:

$$\mathcal{R}_{S_{T_1}, S_{T_2}, S_R}^{\text{non-causal}} \triangleq \bigcup \left\{ (R_1, R_2) : \begin{array}{l} R_1 \leq \max_{p(X_1|S_{T_1})p(X_2|S_{T_2})} I(X_1; Y, S_R | X_2) - I(X_1; S_{T_1}) \\ R_2 \leq \max_{p(X_1|S_{T_1})p(X_2|S_{T_2})} I(X_2; Y, S_R | X_1) - I(X_2; S_{T_2}) \\ R_1 + R_2 \leq \max_{p(X_1|S_{T_1})p(X_2|S_{T_2})} \left\{ \begin{array}{l} I(X_1, X_2; Y, S_R) \\ -I(X_1; S_{T_1}) - I(X_2; S_{T_2}) \end{array} \right\} \end{array} \right\}. \quad (3)$$

<sup>2</sup>Here, we consider the cases where the side information of two transmitters are independent. Note this is reasonable when PU signal power is relatively low and the cognitive transmitters keep a non-trivial distance with each other.

### 3) Strong Spectrum Sensing Capability at the Receiver.

*Theorem 3:* For the three-switch MAC channel, if the transmitters' side information  $S_{T_1}$  and  $S_{T_2}$  are known to the receiver, i.e.  $(S_{T_1}, S_{T_2}) = f(S_R)$ , no matter whether the transmitters' side information is *causal or non-causal*, the channel capacity regions are the same, as given by:

$$C_{(S_{T_1}, S_{T_2})=f(S_R)}^{\text{causal, non-causal}} \triangleq \bigcup \left\{ (R_1, R_2) : \begin{array}{l} R_1 \leq \max_{p(X_1|S_{T_1})p(X_2|S_{T_2})} I(X_1; Y, S_R | X_2) \\ R_2 \leq \max_{p(X_1|S_{T_1})p(X_2|S_{T_2})} I(X_2; Y, S_R | X_1) \\ R_1 + R_2 \leq \max_{p(X_1|S_{T_1})p(X_2|S_{T_2})} I(X_1, X_2; Y, S_R) \end{array} \right\}. \quad (4)$$

### B. Outer bounds and Inner bounds for Gaussian Switch Channel

Since the achievable rate regions derived are intractable, we further explore explicit outer and inner bounds of the capacity region with the help of additional side information. We restrict to the Gaussian case to obtain the optimal results.

*Definition 1:* To facilitate the analysis, we define six events ( $a$  to  $f$ ) related to primary states  $S_{T_1}$ ,  $S_{T_2}$  and  $S_R$ , together with their probabilities of occurrence as follows:

$$\begin{aligned} a : \{S_R = S_{T_1} = 1, S_{T_2} = 0\}, \quad p_a &\triangleq p(S_R = S_{T_1} = 1, S_{T_2} = 0); \\ b : \{S_R = S_{T_2} = 1, S_{T_1} = 0\}, \quad p_b &\triangleq p(S_R = S_{T_2} = 1, S_{T_1} = 0); \\ c : \{S_R = S_{T_1} = S_{T_2} = 1\}, \quad p_c &\triangleq p(S_R = S_{T_1} = S_{T_2} = 1); \\ d : \{S_{T_1} = 1, S_{T_2} = 0\}, \quad p_d &\triangleq p(S_{T_1} = 1, S_{T_2} = 0); \\ e : \{S_{T_1} = 0, S_{T_2} = 1\}, \quad p_e &\triangleq p(S_{T_1} = 0, S_{T_2} = 1); \\ f : \{S_{T_1} = S_{T_2} = 1\}, \quad p_f &\triangleq p(S_{T_1} = S_{T_2} = 1). \end{aligned}$$

*Theorem 4:* For the three-switch MAC channel, *outer bound 1* of the capacity region can be obtained by assuming global side information at both the transmitters and the receiver:

$$C_{*,*,*}(P_1, P_2) = \bigcup_{\substack{p_a P_1^a + p_c P_1^c \leq P_1 \\ p_b P_2^b + p_c P_2^c \leq P_2}} \left\{ (R_1, R_2) : \begin{array}{l} R_1 \leq p_a \log(1 + P_1^a) + p_c \log(1 + P_1^c) \\ R_2 \leq p_b \log(1 + P_2^b) + p_c \log(1 + P_2^c) \\ R_1 + R_2 \leq \left( p_a \log(1 + P_1^a) + p_b \log(1 + P_2^b) \right. \\ \quad \left. + p_c \log(1 + P_1^c + P_2^c) \right) \end{array} \right\}. \quad (5)$$

*Theorem 5:* For the three-switch MAC channel, *outer bound 2* of the capacity region can be obtained by assuming full side information only at the receiver:

$$C_{S_{T_1}, S_{T_2}, *}(P_1, P_2) = \bigcup_{\substack{P_1^d = P_1^f \leq \frac{P_1}{p_d + p_f} \\ P_2^e = P_2^f \leq \frac{P_2}{p_e + p_f}}} \left\{ (R_1, R_2) : \begin{array}{l} R_1 \leq p_a \log(1 + P_1^d) + p_c \log(1 + P_1^f) \\ R_2 \leq p_b \log(1 + P_2^e) + p_c \log(1 + P_2^f) \\ R_1 + R_2 \leq p_a \log(1 + P_1^d) + p_b \log(1 + P_2^e) \\ \quad + p_c \log(1 + P_1^f + P_2^f) \end{array} \right\}. \quad (6)$$

*Theorem 6:* For the three-switch MAC channel (with either causal or non-causal side information), an *inner bound* of the capacity region can be obtained as follows:

$$C_{S_{T_1}, S_{T_2}, S_R}^{\text{inner}} = \bigcup_{\substack{P_1^d = P_1^f \leq \frac{P_1}{p_d + p_f} \\ P_2^e = P_2^f \leq \frac{P_2}{p_e + p_f}}} \left\{ (R_1, R_2) : \begin{array}{l} R_1^* - \Delta R_1 \\ R_2^* - \Delta R_2 \\ R_1^* + R_2^* - \Delta(R_1 + R_2) \end{array} \right\}, \quad (7)$$

where  $(R_1^*, R_2^*)$  denotes the rate pair in *outer bound 2*, and  $0 \leq \Delta R_1 \leq p_a H(S_{T_1} | S_R)$ ,  $0 \leq \Delta R_2 \leq p_b H(S_{T_2} | S_R)$ , and  $0 \leq \Delta(R_1 + R_2) \leq p_c H(S_{T_1}, S_{T_2} | S_R)$  denote the rate gaps.

*Remarks:* When the PU states change very slowly, or the states among transmitters and receiver are highly correlated, we show that the outer bounds and inner bound coincide. In the case with global side information, we can employ rate and power allocation strategies to improve the system's overall performance. Specifically, if we impose power constraints on both transmitters, an optimal rate and power allocation scheme is obtained to maximize the sum rate. We also analyze the effect of correlation in side information and PU occupation probability on the sum rate.

## IV. ANALYSIS

### A. Preliminaries

The memoryless cognitive MAC channel is modeled as a three-switch MAC channel, so that existing results on the memoryless MAC channel with transmitter and receiver side information [16] can be employed to derive the cognitive rate regions, which are listed as the following lemmas:

*Lemma 1. Causal Case:* An achievable rate region of the discrete memoryless MAC channel with receiver side information and independent causal transmitter side information is given by



the convex closure of the rate pairs satisfying:

$$\bigcup_{p_{\text{causal}}} \left\{ (R_1, R_2) : \begin{array}{l} R_1 \leq I(U_1; Y, S_R | U_2) = I(U_1; Y | U_2, S_R) \\ R_2 \leq I(U_2; Y, S_R | U_1) = I(U_2; Y | U_1, S_R) \\ R_1 + R_2 \leq I(U_1, U_2; Y, S_R) = I(U_1, U_2; Y | S_R) \end{array} \right\}, \quad (8)$$

where the message is contained in the mutually independent auxiliary random variables  $U_1$  and  $U_2$ , and the causality is embodied in the following conditional probability distribution:

$$p_{\text{causal}} = \left\{ \begin{array}{l} p(U_1, U_2, X_1, X_2 | S_{T_1}, S_{T_2}) \\ = p(U_1, X_1 | S_{T_1}) p(U_2, X_2 | S_{T_2}) \\ = p(U_1) p(X_1 | U_1, S_{T_1}) p(U_2) p(X_2 | U_2, S_{T_2}) \end{array} \right\}.$$

*Lemma 2. Non-causal Case:* An achievable rate region of the discrete memoryless MAC channel with receiver side information and independent non-causal transmitter side information is given by the convex closure of the rate pairs satisfying:

$$\bigcup_{p_{\text{non-causal}}} \left\{ (R_1, R_2) : \begin{array}{l} R_1 \leq I(U_1; Y, S_R | U_2) - I(U_1; S_{T_1}) \\ R_2 \leq I(U_2; Y, S_R | U_1) - I(U_2; S_{T_2}) \\ R_1 + R_2 \leq I(U_1, U_2; Y, S_R) - I(U_1; S_{T_1}) - I(U_2; S_{T_2}) \end{array} \right\}, \quad (9)$$

where the message is contained in the mutually independent auxiliary random variables  $U_1$  and  $U_2$ , and the non-causality is embodied in the following conditional probability distribution:

$$p_{\text{non-causal}} = \left\{ \begin{array}{l} p(U_1, U_2, X_1, X_2 | S_{T_1}, S_{T_2}) \\ = p(U_1, X_1 | S_{T_1}) p(U_2, X_2 | S_{T_2}) \\ = p(U_1 | S_{T_1}) p(X_1 | U_1, S_{T_1}) p(U_2 | S_{T_2}) p(X_2 | U_2, S_{T_2}) \end{array} \right\}.$$

*Lemma 3.* If the transmitters' side information can be expressed as a function of the receiver side information, i.e.  $\{S_{T_1}, S_{T_2}\} = f(S_R)$ , the memoryless MAC channel capacity regions are the same for causal and non-causal cases, i.e.  $C^{\text{causal}} = C^{\text{non-causal}}$ , given by

$$\bigcup \left\{ (R_1, R_2) : \begin{array}{l} R_1 \leq I(X_1; Y | S_R, X_2) \\ R_2 \leq I(X_2; Y | S_R, X_1) \\ R_1 + R_2 \leq I(X_1, X_2; Y | S_R) \end{array} \right\}, \quad (10)$$

with the conditional probability distribution  $p(X_1, X_2 | S_{T_1}, S_{T_2}) = p(X_1 | S_{T_1}) p(X_2 | S_{T_2})$ .

### B. Achievable Rate and Capacity Regions with Causal and Non-causal Side Information

1) *Proof for Theorem 1:* We know from [20] that the transmitted symbols  $X_1, X_2$  are actually deterministic functions of causal side information  $S_{T_1}, S_{T_2}$  and auxiliary random variables  $U_1, U_2$ :

$$\begin{cases} X_1 = f_1(U_1, S_{T_1}) \\ X_2 = f_2(U_2, S_{T_2}) \end{cases}$$

Since  $S_{T_1}, S_{T_2} \in \{0, 1\}$ , we define

$$X_1 = \begin{cases} g_1(U_1), & \text{for } S_{T_1} = 1, \\ h_1(U_1), & \text{for } S_{T_1} = 0, \end{cases} \quad X_2 = \begin{cases} g_2(U_2), & \text{for } S_{T_2} = 1, \\ h_2(U_2), & \text{for } S_{T_2} = 0. \end{cases}$$

When  $S_{T_1}$  or  $S_{T_2}$  is zero,  $Y$  is not influenced by  $X_1$  or  $X_2$ . Therefore, we can assume

$$\begin{aligned} X_1 &= f_1(U_1), & \text{for } S_{T_1} = 0, 1, \\ X_2 &= f_2(U_2), & \text{for } S_{T_2} = 0, 1. \end{aligned}$$

Note that  $X_1(X_2)$  is the deterministic function of  $U_1(U_2)$ , and the distribution of  $U_1(U_2)$  is independent of  $S_{T_1}(S_{T_2})$ , thus decoding  $X_1(X_2)$  is sufficient for decoding  $U_1(U_2)$ , and  $U_1$  and  $X_1(U_2$  and  $X_2)$  are equivalent in terms of the amount of mutual information. Without loss of generality, the rate region of cognitive MAC channel can be formulated by replacing  $U_1$  and  $U_2$  with  $X_1$  and  $X_2$  respectively in *Lemma 1*. This completes the proof of *Theorem 1*.

2) *Proof for Theorem 2:* We first derive the rate of  $R_1$  based on *Lemma 2*.

$$\begin{aligned} R_1 &= \max_{\substack{p(U_1|S_{T_1})p(X_1|U_1, S_{T_1}) \\ p(U_2|S_{T_2})p(X_2|U_2, S_{T_2})}} I(U_1; Y, S_R | U_2) - I(U_1; S_{T_1}) \\ &\stackrel{(a)}{=} \max_{\substack{p(U_1|S_{T_1})p(X_1|U_1) \\ p(U_2|S_{T_2})p(X_2|U_2)}} I(U_1, X_1; Y, S_R | U_2, X_2) - I(U_1, X_1; S_{T_1}) \\ &= \max_{\substack{p(U_1|S_{T_1})p(X_1|U_1) \\ p(U_2|S_{T_2})p(X_2|U_2)}} \left( \begin{aligned} &I(X_1; Y, S_R | U_2, X_2) + I(U_1; Y, S_R | X_1, U_2, X_2) \\ &- (I(X_1; S_{T_1}) + I(U_1; S_{T_1} | X_1)) \end{aligned} \right) \\ &\stackrel{(b)}{=} \max_{\substack{p(U_1|S_{T_1})p(X_1|U_1)}} \left( \begin{aligned} &I(X_1; Y, S_R | X_2) - I(X_1; S_{T_1}) \\ &+ I(U_1; Y, S_R | X_1) - I(U_1; S_{T_1} | X_1) \end{aligned} \right) \\ &\stackrel{(c)}{\leq} \max_{p(U_1|S_{T_1})p(X_1|U_1)} I(X_1; Y, S_R | X_2) - I(X_1; S_{T_1}) \end{aligned}$$

$$\leq \max_{p(X_1|S_{T_1})} I(X_1; Y, S_R | X_2) - I(X_1; S_{T_1}),$$

where (a) is due to  $X_1 = f_1(U_1)$ , for  $S_{T_1} = 0, 1$ , and  $X_2 = f_2(U_2)$ , for  $S_{T_2} = 0, 1$ ;

(b) holds as  $S_{T_1}$  and  $S_{T_2}$  are independent, thus  $U_2, X_2$  are independent of  $U_1, X_1$ ;

(c) is obtained as  $(U_1 | X_1 = x_1) \rightarrow (S_{T_1} | X_1 = x_1) \rightarrow (Y, S_R | X_1 = x_1)$  forms a Markov Chain.

Therefore  $I(U_1; Y, S_R | X_1) - I(U_1; S_{T_1} | X_1) \leq 0$ , and the equality is achievable by setting  $U_1 = X_1$ .

By symmetry we can similarly obtain  $R_2 \leq \max_{p(X_2|S_{T_2})} I(X_2; Y, S_R | X_1) - I(X_2; S_{T_2})$ .

For the sum rate of  $R_1 + R_2$ , we have [16]

$$R_1 + R_2 = \max_{\substack{p(U_1|S_{T_1})p(X_1|U_1, S_{T_1}) \\ p(U_2|S_{T_2})p(X_2|U_2, S_{T_2})}} I(U_1, U_2; Y, S_R) - I(U_1; S_{T_1}) - I(U_2; S_{T_2}) \quad (11)$$

$$\begin{aligned} &\stackrel{(a)}{=} \max_{\substack{p(U_1|S_{T_1})p(X_1|U_1) \\ p(U_2|S_{T_2})p(X_2|U_2)}} I(U_1, X_1, U_2, X_2; Y, S_R) - I(U_1, X_1; S_{T_1}) - I(U_2, X_2; S_{T_2}) \\ &= \max_{\substack{p(U_1|S_{T_1})p(X_1|U_1) \\ p(U_2|S_{T_2})p(X_2|U_2)}} \left( \begin{aligned} &I(X_1, X_2; Y, S_R) + I(U_1, U_2; Y, S_R | X_1, X_2) \\ &- (I(X_1; S_{T_1}) + I(U_1; S_{T_1} | X_1)) \\ &- (I(X_2; S_{T_2}) + I(U_2; S_{T_2} | X_2)) \end{aligned} \right) \\ &\stackrel{(b)}{=} \max_{\substack{p(U_1|S_{T_1})p(X_1|U_1) \\ p(U_2|S_{T_2})p(X_2|U_2)}} \left( \begin{aligned} &I(X_1, X_2; Y, S_R) - I(X_1; S_{T_1}) - I(X_2; S_{T_2}) \\ &+ I(U_1, U_2; Y, S_R | X_1, X_2) - I(U_1; S_{T_1} | X_1, X_2) - I(U_2; S_{T_2} | X_1, X_2) \end{aligned} \right) \\ &\stackrel{(c)}{=} \max_{\substack{p(U_1|S_{T_1})p(X_1|U_1) \\ p(U_2|S_{T_2})p(X_2|U_2)}} \left( \begin{aligned} &I(X_1, X_2; Y, S_R) - I(X_1; S_{T_1}) - I(X_2; S_{T_2}) \\ &+ I(U_1, U_2; Y, S_R | X_1, X_2) - I(U_1, U_2; S_{T_1}, S_{T_2} | X_1, X_2) \end{aligned} \right) \\ &\stackrel{(d)}{\leq} \max_{\substack{p(U_1|S_{T_1})p(X_1|U_1) \\ p(U_2|S_{T_2})p(X_2|U_2)}} (I(X_1, X_2; Y, S_R) - I(X_1; S_{T_1}) - I(X_2; S_{T_2})) \\ &\leq \max_{p(X_1|S_{T_1})p(X_2|S_{T_2})} (I(X_1, X_2; Y, S_R) - I(X_1; S_{T_1}) - I(X_2; S_{T_2})), \quad (12) \end{aligned}$$

where (a) is due to  $X_i = f_i(U_i)$ , for  $S_{T_i} = 0, 1$ ,  $i = 1, 2$ ;

(b) and (c) hold as  $S_{T_1}$  and  $S_{T_2}$  are independent, thus  $U_1, X_1$  are independent of  $U_2, X_2$ ;

(d) is obtained as  $(U_1 | X_1 = x_1, X_2 = x_2) \rightarrow (S_{T_1} | X_1 = x_1, X_2 = x_2) \rightarrow (Y, S_R | X_1 = x_1, X_2 = x_2)$  forms a Markov chain, so does  $(U_2 | X_1 = x_1, X_2 = x_2) \rightarrow (S_{T_2} | X_1 = x_1, X_2 = x_2) \rightarrow$

$(Y, S_R|X_1 = x_1, X_2 = x_2)$ . Hence,  $(U_1, U_2|X_1 = x_1, X_2 = x_2) \rightarrow (S_{T_1}, S_{T_2}|X_1 = x_1, X_2 = x_2) \rightarrow (Y, S_R|X_1 = x_1, X_2 = x_2)$  also forms a Markov Chain. Therefore,  $I(U_1, U_2; Y, S_R|X_1, X_2) - I(U_1, U_2; S_{T_1}, S_{T_2}|X_1, X_2) \leq 0$ . Finally, the equality in (12) is achievable by setting  $U_1 = X_1$  and  $U_2 = X_2$  in (11). This completes the proof of *Theorem 2*.

3) *Proof for Theorem 3*: Since the three-switch MAC channel is a special case of the memoryless channel with side information, only that the side information here is binary. We can directly employ the result of *Lemma 3* to complete the proof.

*Remarks*: In practise, the assumption in *Theorem 3* implies that CH (or BS) can not only sense the primary activities at its side, but also the primary activities near the transmitters. This assumption is reasonable, since CHs or BSs are usually equipped with more powerful hardware and abundant energy supplies.

### C. Outer and Inner Bounds

Since the rate regions derived are intractable in general, we further explore explicit outer and inner bounds of the capacity region with the help of additional side information. We restrict to the Gaussian case to obtain the optimal results [17].

We consider two kinds of additional side information at the cognitive transmitters and/or receiver, and derive the corresponding outer bounds. In *Case 1*, both the transmitters and the receiver have full knowledge of all side information. In *Case 2*, only the receiver has full side information. For the case when only the transmitters (both) have full side information, either of them transmits only when its own switch is on and  $S_R = 1$ . We assume that the receiver can infer the states of transmitters through signal detection, thus this case coincides with *Case 1*. Note that when the receiver also has transmitter side information, there is no differences between the causal and non-causal case [17]. Also, arbitrary correlation among side information may be considered in these genie-aided scenarios.

1) *Outer bounds 1 - Global Side Information*: With global side information, the transmissions occur when  $S_R = 1$  and  $S_{T_1} + S_{T_2} \geq 1$ , which can be further categorized into three subcases.

When  $S_R = S_{T_1} = 1, S_{T_2} = 0$ , the MAC channel degrades into a point-to-point channel, in which Gaussian input is optimal. Assuming CT1's transmission power to be  $P_1^a$ , the achievable rate region corresponding to event  $a$  is:

$$C_{a,*,*,*}(P_1^a, 0) = \bigcup \left\{ \begin{array}{l} R_1^a \leq \log(1 + P_1^a) \\ R_2^a = 0 \end{array} \right\}.$$

Similarly, when  $S_R = S_{T_2} = 1, S_{T_1} = 0$ , assuming CT2's transmission power to be  $P_2^b$ , the achievable rate region corresponding to event  $b$  is:

$$C_{b,*,*,*}(0, P_2^b) = \bigcup \left\{ \begin{array}{l} R_1^b = 0 \\ R_2^b \leq \log(1 + P_2^b) \end{array} \right\}.$$

When  $S_R = S_{T_1} = S_{T_2} = 1$ , the channel is the traditional MAC channel. Assuming the transmission power of CT1 and CT2 to be  $P_1^c$  and  $P_2^c$ , respectively, the achievable rate region corresponding to event  $c$  is:

$$C_{c,*,*,*}(P_1^c, P_2^c) = \bigcup \left\{ \begin{array}{l} R_1^c \leq \log(1 + P_1^c) \\ R_2^c \leq \log(1 + P_2^c) \\ R_1^c + R_2^c \leq \log(1 + P_1^c + P_2^c) \end{array} \right\}.$$

Taking into account the power constraints:  $p_a P_1^a + p_c P_1^c \leq P_1$ , and  $p_b P_2^b + p_c P_2^c \leq P_2$ , *outer bounds 1* can be derived as in (5).

*Optimal Rate/Power Allocation:* When both the transmitters and receiver have global state information, we can further explore the optimal rate and power allocation.

Without loss of generality, we may take the optimal sum rate as our objective:

$$\begin{aligned} & \text{maximize : } R_1 + R_2 = p_a \log(1 + P_1^a) + p_b \log(1 + P_2^b) + p_c \log(1 + P_1^c + P_2^c), \\ & \text{subject to : } \begin{cases} p_a P_1^a + p_c P_1^c \leq P_1, \\ p_b P_2^b + p_c P_2^c \leq P_2, \\ P_1^a, P_2^b, P_1^c, P_2^c \geq 0. \end{cases} \end{aligned} \quad (13)$$

This is a convex optimization problem, and can be solved through KKT conditions.

$$\begin{cases} \frac{\partial L(P_1^a, P_2^b, P_1^c, P_2^c)}{\partial P_1^a} = 0, \\ \frac{\partial L(P_1^a, P_2^b, P_1^c, P_2^c)}{\partial P_2^b} = 0. \end{cases}$$

Substituting the constraints  $P_1^c = \frac{P_1 - p_a P_1^a}{p_c}$ ,  $P_2^c = \frac{P_2 - p_b P_2^b}{p_c}$ , the solution is obtained as

$$\begin{aligned} P_1^a &= P_2^b = \frac{P_1 + P_2}{p_a + p_b + p_c}, \\ P_1^c &= \frac{(p_b + p_c)P_1 - p_a P_2}{(p_a + p_b + p_c)p_c}, \\ P_2^c &= \frac{(p_a + p_c)P_2 - p_b P_1}{(p_a + p_b + p_c)p_c}. \end{aligned} \quad (14)$$

The corresponding optimal sum rate is given as follows.

*Corollary 1:* The maximal sum rate is  $(R_1 + R_2)_{\max} = (p_a + p_b + p_c) \log \left( 1 + \frac{P_1 + P_2}{p_a + p_b + p_c} \right)$ .

2) *Outer bounds 2 - Full Side Information at Receiver:* In Case 2, transmissions occur only when  $S_{T_1} + S_{T_2} \geq 1$ . This can be further categorized into three subcases, which correspond to the events  $d$ ,  $e$  and  $f$  in Definition 1.

When  $S_{T_1} = 1, S_{T_2} = 0$  and  $S_{T_1} = 0, S_{T_2} = 1$ , the MAC channel degrades into point-to-point channels. We assume CT1 and CT2's transmission power to be  $(P_1^d, 0)$  and  $(0, P_1^e)$ , respectively.

When  $S_R = 1$ , the achievable rate regions for event  $d$  and  $e$  are:

$$C_{d, S_{T_1}, S_{T_2}, *}(P_1^d, 0) = \bigcup \left\{ \begin{array}{l} R_1^d \leq \log(1 + P_1^d) \\ R_2^d = 0 \end{array} \right\},$$

$$C_{e, S_{T_1}, S_{T_2}, *}(0, P_2^e) = \bigcup \left\{ \begin{array}{l} R_1^e = 0 \\ R_2^e \leq \log(1 + P_2^e) \end{array} \right\}.$$

When  $S_{T_1} = S_{T_2} = 1$ , the channel becomes the basic MAC channel. We assume the transmission power of CT1 and CT2 to be  $P_1^f$  and  $P_2^f$ , respectively. When  $S_R = 1$ , the achievable rate region for event  $f$  is:

$$C_{f, S_{T_1}, S_{T_2}, *}(P_1^f, P_2^f) = \bigcup \left\{ \begin{array}{l} R_1^f \leq \log(1 + P_1^f) \\ R_2^f \leq \log(1 + P_2^f) \\ R_1^f + R_2^f \leq \log(1 + P_1^f + P_2^f) \end{array} \right\}.$$

The power constraints are also considered:

$$\begin{cases} p_d P_1^d + p_f P_1^f \leq P_1, \\ p_e P_2^e + p_f P_2^f \leq P_2. \end{cases}$$

Since the transmitters do not have full side information, they can not discriminate the three subcases and have to use the same transmission power, i.e.  $P_1^d = P_1^f, P_2^e = P_2^f$ . The transmission power is bounded as  $P_1^d = P_1^f \leq \frac{P_1}{p_d + p_f}$  and  $P_2^e = P_2^f \leq \frac{P_2}{p_e + p_f}$ .

Although transmissions occur with the probabilities of  $p_d, p_e$  and  $p_f$ , effective transmissions occur only when  $S_R = 1$ , with the probabilities of  $p_a, p_b$  and  $p_c$ . Only effective transmissions contribute to system capacity, therefore *outer bound 2* is derived as in (6).

3) *Inner bound*: The inner bound can not be calculated directly. However, we can obtain it with the help of Genie information [17]. Suppose a genie provides some additional information about the transmitter states to the receiver every channel use through a genie variable  $G$ . The idea of genie-assisted lowerbound is that ‘the improvement in capacity induced by the genie information  $G$  cannot exceed the entropy rate of the genie information itself’, i.e.,  $R_{S_{T_1}, S_{T_2}, (S_R+G)} - R_{S_{T_1}, S_{T_2}, S_R} \leq H(G|S_R)$ .

When  $S_{T_1} = 1, S_{T_2} = 0$  or  $S_{T_1} = 0, S_{T_2} = 1$ , the MAC channel degrades to a point-to-point channel. Following the idea of genie-assisted bound, the inner bounds for the two subcases are:

$$R_1 \geq R_1^* - H(G_1|S_R), \quad R_2 \geq R_2^* - H(G_2|S_R).$$

When  $S_{T_1} = 1, S_{T_2} = 1$ , the lowerbound on sum rate is:

$$R_1 + R_2 \geq R_1^* + R_2^* - H(G_1, G_2|S_R),$$

where  $(R_1^*, R_2^*)$  denotes the maximal rate pair in *outer bound 2*. To sum up,  $\Delta R_1 \leq p_a H(G_1|S_R) \leq p_a H(S_{T_1}|S_R)$ ,  $\Delta R_2 \leq p_b H(G_2|S_R) \leq p_b H(S_{T_2}|S_R)$ , and  $\Delta(R_1 + R_2) \leq p_c H(G_1, G_2|S_R) \leq p_c H(S_{T_1}, S_{T_2}|S_R)$ . Thus, the overall inner bound is obtained as in *Theorem 6*.

*Remarks*: According to *Theorem 6*, in event  $d$ ,  $e$  and  $f$ , the inner bounds are  $H(G_1|S_R)$ ,  $H(G_2|S_R)$  and  $H(G_1, G_2|S_R)$  bits per channel use lower than the outer bounds with full side information at the receiver, respectively.

The transmitters only need to send one bit notification to the receiver when there is a change of PU states. We assume that, on average, the PU activities at the transmitters change every  $N$  time slots. Then the genie information rate is at most  $1/N$  bit per time slot for each transceiver pair. Formally, the gap between *outer bound 2* and *inner bound* is restricted as  $H(G_1|S_R) = H(G_2|S_R) \leq H(G_1) = H(G_2) = \frac{1}{N}$  and  $H(G_1, G_2|S_R) \leq H(G_1, G_2) = \frac{2}{N}$ .

Moreover, when the side information between the transmitters and receiver are highly correlated,  $H(S_{T_1}|S_R)$ ,  $H(S_{T_2}|S_R)$  and  $H(S_{T_1}, S_{T_2}|S_R)$  also converge to zero, and so do  $H(G_1|S_R)$ ,  $H(G_2|S_R)$  and  $H(G_1, G_2|S_R)$ . Therefore, we conclude that the outer bounds and inner bound coincide when the states of PU signals change very slowly or the side information between the transmitters and receiver are highly correlated.

#### D. Effect of correlation and PU occupation rate

We take the optimized sum rate in *Corollary 1* as a criterion, and analyze how it is influenced by two system parameters within a specific probabilistic model. We assume the PU occupation rates at both transmitters and the receiver to be the same,  $\mu = 1 - E[S_{T_1}] = 1 - E[S_{T_2}] = 1 - E[S_R]$ ; and the mutual correlation coefficient between the three states also to be the same, as  $\rho$ . According to the definition of correlation coefficient, the joint probability distribution is denoted in Table I, where  $p_0 = \mu[\mu^2 + \rho(1 - \mu^2)]$ ,  $p_1 = (1 - \rho^2)(1 - \mu)\mu^2$ ,  $p_2 = (1 - \rho)\mu[(1 - \mu)^2 + \rho(\mu - \mu^2)]$ , and  $p_3 = [(1 - \mu)^2 + \rho(\mu - \mu^2)](1 - \mu + \mu\rho)$ . The probabilities for the six events in *Definition 1* can be expressed as Thus, the probabilities for the six events can be expressed as functions of  $\mu$  and  $\rho$ :

$$\begin{cases} p_c = p_3, \\ p_a = p_b = p_2, \\ p_f = p_2 + p_3, \\ p_d = p_e = p_1 + p_2. \end{cases} \quad (15)$$

Combining (14) and (15), the sum rate can be re-written in the forms of  $\rho$  and  $\mu$ :

$$(R_1 + R_2)_{\max} = p(\mu, \rho) \log \left( 1 + \frac{P_1 + P_2}{p(\mu, \rho)} \right), \quad (16)$$

where  $p(\mu, \rho) = (1 + \mu - \mu\rho)[(1 - \mu)^2 + \rho(\mu - \mu^2)]$ .

We find that (16) is a monotonically increasing function of  $p(\mu, \rho)$ . Moreover, since  $0 < \mu, \rho < 1$ ,

$$\begin{aligned} \frac{\partial p(\mu, \rho)}{\partial \mu} &= \mu(1 - \rho)^2(3\mu - 2) - 1 < 0, \\ \frac{\partial p(\mu, \rho)}{\partial \rho} &= 2\mu^2(1 - \rho)(1 - \mu) > 0. \end{aligned}$$

Thus (16) is a monotonically decreasing function of  $\mu$  and monotonically increasing function of  $\rho$ . The insights here are that the sum rate increases when the PU is less active and when the correlation among the side information is stronger.

#### E. Extension to Fading and Interference model

We continue to explore a general cognitive MAC channel model to incorporate channel fading and interference. In contrast to the above study, we allow the receiver to stay on all the time since the receiver will not cause any interference to the PU system. Therefore, the receiver needs to decode the SU signal even under severe fading and interference, and the transmitters need



to perform more complex rate/power allocation scheme. Our purpose is to explore how much the performance can be improved with increased processing complexity. However, we show that the performance improvement is only marginal and thus the simplification to the three-switch channel model is worthwhile.

The system model of the cognitive MAC channel with fading and interference is expressed as:

$$Y = S_{T_1} H_1 X_1 + S_{T_2} H_2 X_2 + S_R Z, \quad (17)$$

where  $S_{T_1}$  and  $S_{T_2}$  are the PU states at the transmitters, as defined previously.  $S_R Z$  is the general PU interference at the receiver.  $H_1$  and  $H_2$  are the general channel gains.

By letting  $h_1 = \frac{S_{T_1} H_1}{S_R}$  and  $h_2 = \frac{S_{T_2} H_2}{S_R}$ , the channel model can be normalized as:

$$Y = h_1 X_1 + h_2 X_2 + Z.$$

Here we assume the same distributions for  $h_1$  and  $h_2$  and the same average power constraints for the two transmitters.

Define  $\mathbf{h} = [h_1, h_2]$  as the normalized channel gain vector and consider all fading possibilities, the achievable rate region of the cognitive MAC fading channel [21] can be expressed as:

$$C^{\text{fading}}(P_1, P_2) = \bigcup_{\mathcal{P} \in \mathcal{F}} \left\{ \begin{array}{l} R_1 \leq \sum_{h_1} [p(\mathbf{h}) \log(1 + h_1 P_1(\mathbf{h}))] \\ R_2 \leq \sum_{h_2} [p(\mathbf{h}) \log(1 + h_2 P_2(\mathbf{h}))] \\ R_1 + R_2 \leq \sum_{\mathbf{h}} \left[ p(\mathbf{h}) \log \left( 1 + \sum_{h_i} h_i P_i(\mathbf{h}) \right) \right] \end{array} \right\}. \quad (18)$$

where  $\mathcal{F} \equiv \{\mathcal{P} : E_{\mathbf{h}}[P_i(\mathbf{h})] \leq P_i, \forall i \in 1, 2\}$  is the set of all possible power allocation schemes within the power constraints.

Now, we move on to discuss the optimal rate/power allocation under fading and interference when the PU states and channel gains are known to both the transmitters and receiver. According to [21], when global state information is available, the max sum rate  $R_1 + R_2$  can be optimized over all possible power allocation schemes. The solution has the form of:

$$\begin{aligned} P_1^*(\mathbf{h}, \lambda) &= \begin{cases} \left( \frac{1}{2\lambda_1} - \frac{1}{h_1} \right)^+, & h_1 \geq \frac{\lambda_1}{\lambda_2} h_2, \\ 0, & \text{else,} \end{cases} \\ P_2^*(\mathbf{h}, \lambda) &= \begin{cases} \left( \frac{1}{2\lambda_2} - \frac{1}{h_2} \right)^+, & h_2 \geq \frac{\lambda_2}{\lambda_1} h_1, \\ 0, & \text{else,} \end{cases} \end{aligned} \quad (19)$$

where the constant vector  $\lambda = [\lambda_1, \lambda_2]$  is determined by the following average power constraints:

$$\begin{aligned} \sum_{h_1} \left[ \left( \frac{1}{2\lambda_1} - \frac{1}{h_1} \right)^+ p \left( h_1 \geq \frac{\lambda_1}{\lambda_2} h_2 \right) p(h_1) \right] &\leq P_1, \\ \sum_{h_2} \left[ \left( \frac{1}{2\lambda_2} - \frac{1}{h_2} \right)^+ p \left( h_2 \geq \frac{\lambda_2}{\lambda_1} h_1 \right) p(h_2) \right] &\leq P_2. \end{aligned}$$

Since  $h_1$  and  $h_2$  are identically distributed,  $\lambda_1 = \lambda_2$  by symmetry. A straightforward observation from (19) is that when the channel gain of one SU transmitter is worse than another, the transmission will be turned off to save power. In other words, the two SU transmitters will transmit simultaneously with equal power only when their channel gains are the same. Therefore, the cognitive MAC channel with fading and interference can be further simplified into the following model:

$$Y = S'_{T_1} X_1 + S'_{T_2} X_2 + S'_R Z, \quad (20)$$

where  $S'_{T_1} = \begin{cases} 1, & S_{T_1} = 1 \text{ and } H_1 \geq H_2, \\ 0, & S_{T_1} = 0 \text{ or } H_1 < H_2, \end{cases}$  and  $S'_{T_2} = \begin{cases} 1, & S_{T_2} = 1 \text{ and } H_1 \leq H_2, \\ 0, & S_{T_2} = 0 \text{ or } H_1 > H_2, \end{cases}$  are the binary transmitter side information, and  $S'_R = \frac{S_R}{\max(H_1, H_2)}$  is the normalized receiver side information. Note that all the above factors have included channel fading and the PU interference.

The difference with the three-switch channel is that here the receiver will not turn off the switch even under fading and PU interference. We assume the normalized receiver side information  $S'_R$  to be either  $S'_R = 1$  or  $S'_R = I$ , which corresponds to without and with fading and interference, respectively. The joint probability of  $p(S'_{T_1}, S'_{T_2}, S'_R)$  is defined similarly to *Definition 1*, except that ' $S_R = 0$ ' is replaced by ' $S'_R = I$ ' and ' $S_{T_1}, S_{T_2}, S_R$ ' are replaced by ' $S'_{T_1}, S'_{T_2}, S'_R$ ', as shown in Table I. For convenience of performance comparison, we set the values of joint probabilities here to be the same as in the three-switch model.

Following the same line in Section IV, the maximal sum rate for cognitive MAC fading channel is obtained after solving the convex optimization problem:

$$\begin{aligned} &(R_1 + R_2)_{\max} \\ &= \max_{P'_1 + P'_2 + P''_1 + P''_2 \leq P_1 + P_2} \left[ p_{\text{non-fading}} \log \left( 1 + \frac{P'_1 + P'_2}{p_{\text{non-fading}}} \right) + p_{\text{fading}} \log \left( 1 + \frac{P''_1 + P''_2}{T^2 p_{\text{fading}}} \right) \right], \quad (21) \end{aligned}$$

where  $p_{\text{non-fading}} \triangleq p_a + p_b + p_c$  and  $p_{\text{fading}} \triangleq p_d + p_e + p_f - (p_a + p_b + p_c)$ .  $P'_1 + P'_2$  is the power spent when  $S'_R = 1$ , and  $P''_1 + P''_2$  is the power spent when  $S'_R = I$ , both of which are under the

average power constraints. It is easy to find that the above maximum is achieved when

$$\begin{aligned} P''_1 + P''_2 &= \left( \frac{(P_1 + P_2)p_{\text{fading}} - (I^2 - 1)p_{\text{non-fading}}p_{\text{fading}}}{p_{\text{non-fading}} + p_{\text{fading}}} \right)^+, \\ P'_1 + P'_2 &= P_1 + P_2 - (P''_1 + P''_2). \end{aligned} \quad (22)$$

From (22) we observe that when the fading or interference is very severe (i.e.  $I$  is large), or the fading and interference probabilities are low, the major proportion of power is spent during non-fading periods. In these cases, the maximal sum rate will be close to that of the three-switch model, in which all power is spent during good channel conditions. The numerical results in Section V also show that the performance improvement contributed by turning on the switch under fading and interference (even when  $I$  is not large) is limited, which further validates the effectiveness of the proposed simplified model.

#### F. Extension to the $m$ -user Case

Throughout this paper, we focus on the three-switch MAC channel where we only consider two transmitters. However, the results and analysis can be generalized to  $m$ -user case. Here we provide a brief discussion. The  $m$ -user model is expressed as

$$Y = \left( \sum_{i=1}^m S_{T_i} X_i + Z \right) S_R. \quad (23)$$

We denote  $M \subseteq \{1, 2, \dots, m\}$  as the set of transmitters, and  $M^c$  as its complement. Let  $R(M) = \sum_{i \in M} R_i$  be the sum rate. Let  $U(M) = \{U_i : i \in M\}$ ,  $X(M) = \{X_i : i \in M\}$  and  $S_T(M) = \{S_{T_i} : i \in M\}$ . The other definitions are all the same as in the two-user case.

The achievable region with independent general side information is expressed as:

$$\bigcup_{p_{\text{causal}}} \{ \mathbf{R} : R(M) \leq I(U(M); Y, S_R | U(M^c)) \},$$

where  $p_{\text{causal}} = \prod_{i=1}^m p(U_i) p(X_i | U_i, S_{T_i})$  for causal case; and

$$\bigcup_{p_{\text{non-causal}}} \left\{ \mathbf{R} : R(M) \leq I(U(M); Y, S_R | U(M^c)) - \sum_{i \in M} I(U_i; S_{T_i}) \right\},$$

where  $p_{\text{non-causal}} = \prod_{i=1}^m p(U_i | S_{T_i}) p(X_i | U_i, S_{T_i})$  for non-causal case.

Following the same lines in the proofs of Theorems 1-2, the achievable region for  $m$ -user cognitive MAC channel with independent on/off side information can be expressed as:

$$\bigcup \left\{ \mathbf{R} : R(M) \leq \max_{\prod_{i \in M} p(X_i)} I(X(M); Y, S_R | X(M^c)) \right\}$$

for causal case, and

$$\bigcup \left\{ \mathbf{R} : R(M) \leq \max_{\prod_{i \in M} p(X_i | S_{T_i})} \left( I(X(M); Y, S_R | X(M^c)) - \sum_{i \in M} I(X_i; M_{T_i}) \right) \right\}$$

for non-causal case.

Following the same lines in the proofs of Theorem 4-6, the outer and inner bounds for  $m$ -user case can be obtained. We define  $\mathbf{S}_T = \{S_{T_1}, S_{T_2}, \dots, S_{T_m}\} \in \mathcal{S}_T$  as the  $m$ -length binary state vector of the  $m$  transmitters, where  $\mathcal{S}_T : |\mathcal{S}_T| = 2^m$  is the set of all possible transmitter states. We also define  $M_{\mathbf{S}_T} = \{i : S_{T_i} = 1 | \mathbf{S}_T\}$  as the set of active transmitters when the state vector is  $\mathbf{S}_T$ .

Correspondingly, *outer bound 1* which assumes global side information is

$$C_m^{outer1}(P_1, P_2, \dots, P_m) = \bigcup_{\mathcal{C}_1} \left\{ \mathbf{R} : R(M) \leq \sum_{\mathbf{S}_T \in \mathcal{S}_T} p(\mathbf{S}_T, S_R = 1) \log \left( 1 + \sum_{i \in M \cap M_{\mathbf{S}_T}} P_i^{(\mathbf{S}_T, S_R=1)} \right) \right\},$$

where  $\mathcal{C}_1$  is the power constraint expressed as

$$\mathcal{C}_1 \triangleq \left\{ \mathcal{P} : \sum_{\mathbf{S}_T \in \mathcal{S}_T} p(\mathbf{S}_T, S_R = 1) P_i^{\mathbf{S}_T} \leq P_i, \forall i \in \{1, 2, \dots, m\} \right\},$$

*outer bound 2* which assumes full side information at the receiver is

$$C_m^{outer2}(P_1, P_2, \dots, P_m) = \bigcup_{\mathcal{C}_2} \left\{ \mathbf{R} : R(M) \leq \sum_{\mathbf{S}_T \in \mathcal{S}_T} p(\mathbf{S}_T, S_R = 1) \log \left( 1 + \sum_{i \in M \cap M_{\mathbf{S}_T}} P_i^{\mathbf{S}_T} \right) \right\},$$

where  $\mathcal{C}_2$  is the power constraint expressed as

$$\mathcal{C}_2 \triangleq \left\{ \mathcal{P} : \sum_{\mathbf{S}_T \in \mathcal{S}_T} p(\mathbf{S}_T) P_i^{\mathbf{S}_T} \leq P_i, \forall i \in \{1, 2, \dots, m\} \right\},$$

and *outer bound 3* is the same as *outer bound 1*.

Similarly, the *inner bound* is obtained as

$$C_m^{inner}(P_1, P_2, \dots, P_m) = \left\{ \mathbf{R} : R(M_{\mathbf{S}_T}) = R^{outer2}(M_{\mathbf{S}_T}) - \Delta R(M_{\mathbf{S}_T}) \right\},$$

where  $R^{outer2}(M_{\mathbf{S}_T})$  is the maximal sum rate of transmitter set  $M_{\mathbf{S}_T}$  in *outer bound 2*, and  $\Delta R(M_{\mathbf{S}_T}) \leq p(\mathbf{S}_T, S_R = 1) H(S_T(M) | S_R)$  is the capacity gap.

## V. NUMERICAL RESULTS

### A. Outer and Inner Bounds

To plot the outer and inner bounds, we travel through all possible power pairs which satisfy the power constraints, and compute the corresponding rate pairs. As for the inner bound, we calculate the gap between *outer bound 2* and *inner bound* and subtract it from *outer bound 2*.

Fig. 3 to Fig. 6 are the *outer bound 1*, *2* and the *inner bound* under different PU occupation rates  $\mu$  and PU states correlation coefficients  $\rho$ . The parameters are  $(\mu = 0.1, \rho = 0)$ ,  $(\mu = 0.1, \rho = 0.9)$ ,  $(\mu = 0.5, \rho = 0)$  and  $(\mu = 0.5, \rho = 0.9)$  for the four figures, respectively. All four cases are under the power constraints of  $P_1 \leq 1$  and  $P_2 \leq 1$ . Based on these parameters, we calculate  $p_a$  to  $p_f$  according to (15), and obtain the bounds.

Moreover, in order to observe the effect of PU activities and correlation on capacity, we plot the sum rate of *outer bound 1* with respect to PU occupation rate  $\mu$  and correlation coefficient  $\rho$  on Fig. 7 to Fig. 8, in which the sum rate corresponds to the corner points in Fig. 3 to Fig. 6.

The insights obtained from these results are summarized below.

- 1) Correlation coefficient  $\rho$ : By comparing Fig. 3 with Fig. 4, and Fig. 5 with Fig. 6, we find gaps between the outer and inner bounds become closer as  $\rho$  grows. On the one hand, *outer bound 2* approaches *outer bound 1* as  $\rho$  grows, as the transmitters in *Case 2* gradually have global side information, and *Case 2* evolves to *Case 1*. On the other hand, *inner bound* approaches *outer bound 2* because  $H(S_{T_i}(t)|S_R(t))$  decreases as  $\rho$  grows. In addition, Fig. 8 also demonstrates that the maximal sum rate increases with  $\rho$ , which means more correlation improves the overall performance.
- 2) PU occupation rate  $\mu$ : By comparing Fig. 3 with Fig. 5, and Fig. 4 with Fig. 6, we observe that as  $\mu$  decreases, the overall system rate grows. This is further verified in Fig. 7. When  $\mu = 0$ , the PUs keep silent, and the cognitive MAC rate region evolves to the traditional MAC rate region. When  $\mu = 1$ , the spectrum is always occupied by PUs, the cognitive MAC rate degrades to zero.
- 3) State changing rate: Fig. 3 to Fig. 6 show that, the inner bound are closer to the outer bounds when the  $N$  increases. This indicates that, in the case where the PU states change slowly, the outer and inner bounds coincide.
- 4) Transmitter Side information: Compared with *Case 1*, the transmitters in *Case 2* lack global

side information. Hence, the big gap between *outer bound 1* and 2 implies the importance of the transmitter side information. Moreover, this gap is larger in Fig. 5 than in Fig. 3, which indicates that the transmitter side information is more valuable in busy channels.

### B. Optimal Rate/Power Allocation

We follow the same parameters as in Section V.A, and plot the sum rate  $R_1 + R_2$  using derivations in Section IV.E. The power allocation results are illustrated in Fig. 10 and Fig. 9. The PU occupation rates for both figures are  $\mu = 0.1$ , while the correlation coefficients  $\rho$  are 0 and 0.5, respectively.

Again we assume the average power constraints are  $P_1 \leq 1$  and  $P_2 \leq 1$ , and the top curve corresponds to the case when maximal power is consumed, i.e.  $p_a P_1^a + p_c P_1^c = 1$  and  $p_b P_2^b + p_c P_2^c = 1$ . Since we choose the same values of  $\mu$  and  $\rho$  for both transmitters and the receiver, the power allocation results are symmetric between the two transmitters, i.e.  $P_1^a = P_2^b$ ,  $P_1^c = P_2^c$ . As shown in Fig. 10 and Fig. 9, different power allocations generate different sum rates. Since (13) is a convex optimization problem, we can always find an optimal power allocation scheme which maximize the sum rate. Note that here the individual powers may exceed 1 because our constraints are imposed on the average power, therefore the peak powers can be higher.

### C. Optimized Sum Rate in Fading and Interference model

We now provide numerical results for the cognitive MAC channel with fading and interference, which is discussed in Section VI.G. As shown in Table I, we assume the average power constraints and joint probabilities that include fading and interference to be the same as in Section V.A. In addition, we assume that the power constraints are  $P_1 \leq 1$  and  $P_2 \leq 1$ , and the channel in the bad state (including fading and interference) has an SNR of  $-10\text{dB}$ , which corresponds to  $I^2 = 10$ .

The maximal sum rate under fading and PU interference with respect to PU occupation rate  $\mu$  and correlation coefficient  $\rho$  are plotted in Fig. 11 and Fig. 12, which are computed according to (15), (21), and (22).

By comparing Fig. 11 with Fig. 7 and Fig. 12 with Fig. 8, we find that their curves are close to each other. The performance gap is more obvious only when the fading or interference probability is very high as  $\mu = 0.9$  (note that  $p(S'_R = I) = \mu$ ). The insight is when the bad

channel state lasts for a relatively larger proportion of time, the rate loss from shutting off the receiver during bad channel conditions becomes more significant. However, their overall performances are almost the same, which indicates that this general model can be well replaced by the much simplified three-switch model.

## VI. CONCLUSION

In this paper, the cognitive MAC channel is modeled as a three-switch channel, and the achievable rate regions are formulated by viewing PU activities as causal/non-causal on/off side information. The closed form outer and inner bounds are derived, which are shown to be tight under some special cases. An optimal rate allocation scheme is also proposed to maximize the sum rate with global side information, and the effect of correlation in side information and PU activities is also analyzed. The extension to the fading scenario and general  $m$ -user case discussed. The numerical results show the importance of transmitter side information in enhancing the capacity and the effectiveness of our rate allocation scheme.

## VII. ACKNOWLEDGEMENT

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TABLE I  
JOINT PROBABILITY DISTRIBUTION UNDER NON-FADING AND FADING MODELS

$S_R = 0$ or $S'_R = I$		
$S_{T_1} (S'_{T_1}) \backslash S_{T_2} (S'_{T_2})$	0	1
0	$p_0$	$p_1$
1	$p_1$	$p_2$
$S_R = 1$ or $S'_R = 1$		
$S_{T_1} (S'_{T_1}) \backslash S_{T_2} (S'_{T_2})$	0	1
0	$p_1$	$p_2$
1	$p_2$	$p_3$

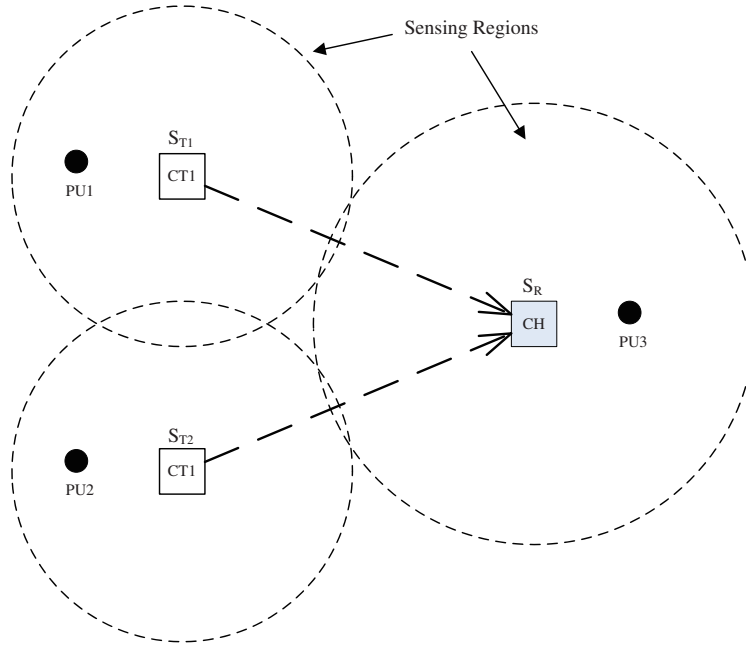


Fig. 1. Memoryless Cognitive MAC Channel

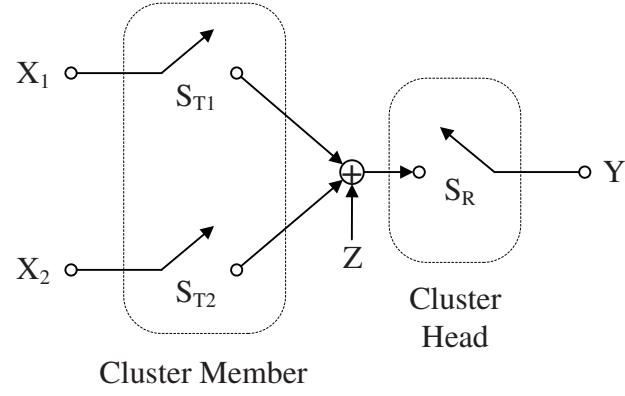


Fig. 2. Three Switch MAC Channel

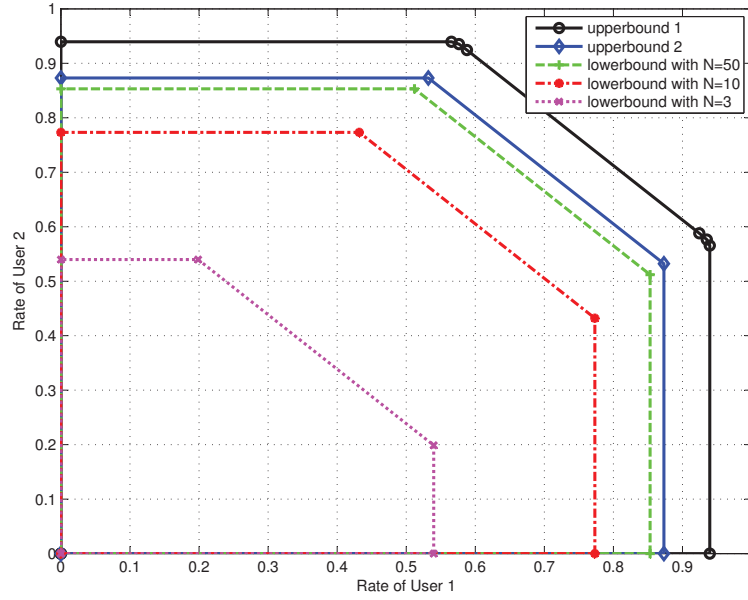


Fig. 3. Capacity region bounds when  $\mu = 0.1, P = 1, \rho = 0$

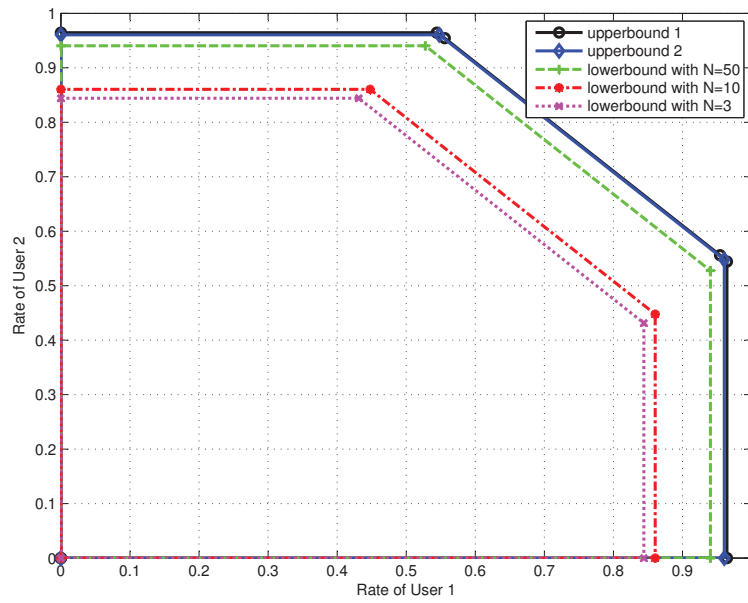


Fig. 4. Capacity region bounds when  $\mu = 0.1$ ,  $P = 1$ ,  $\rho = 0.9$

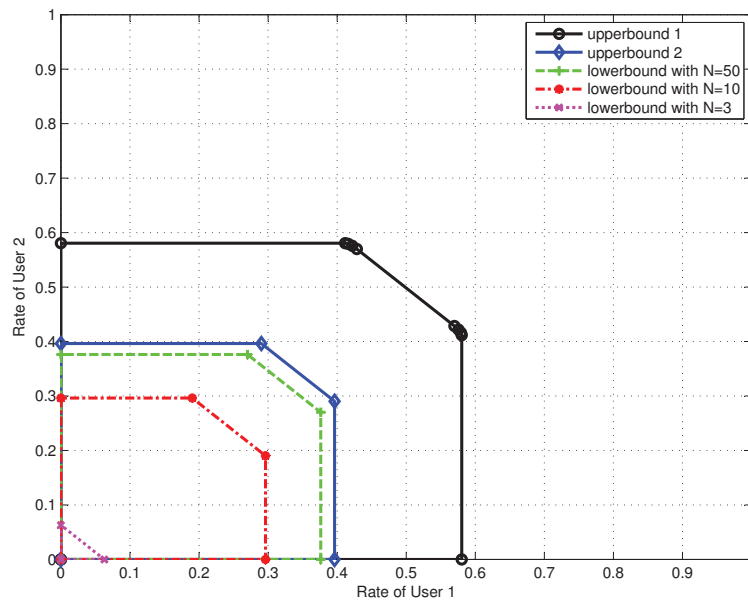


Fig. 5. Capacity region bounds when  $\mu = 0.5$ ,  $P = 1$ ,  $\rho = 0$

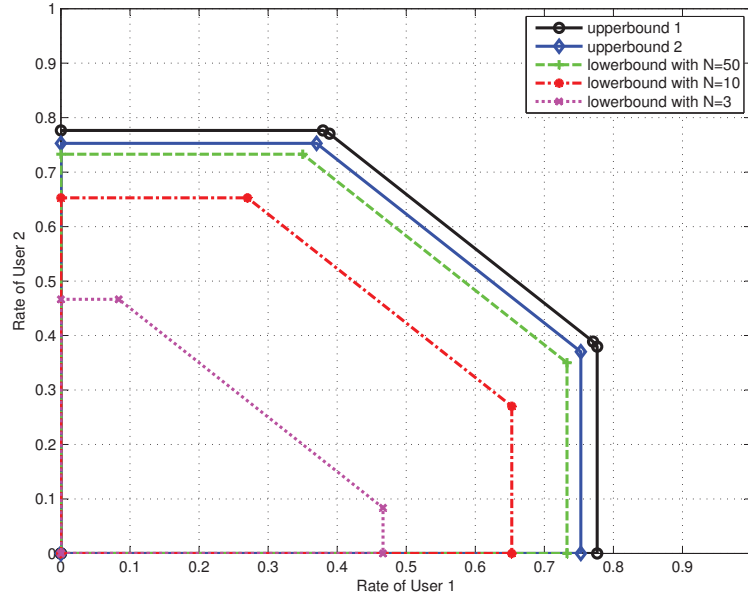


Fig. 6. Capacity region bounds when  $\mu = 0.5$ ,  $P = 1$ ,  $\rho = 0.9$

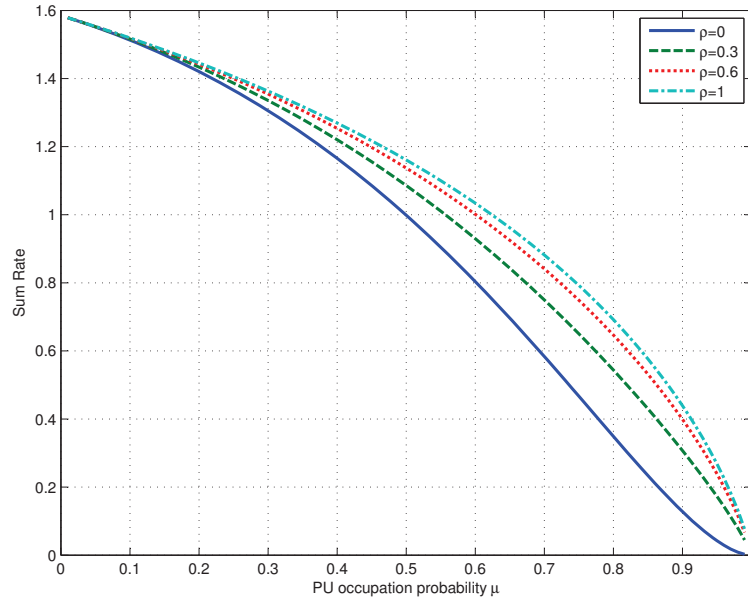


Fig. 7. Effect of PU activities on sum rate

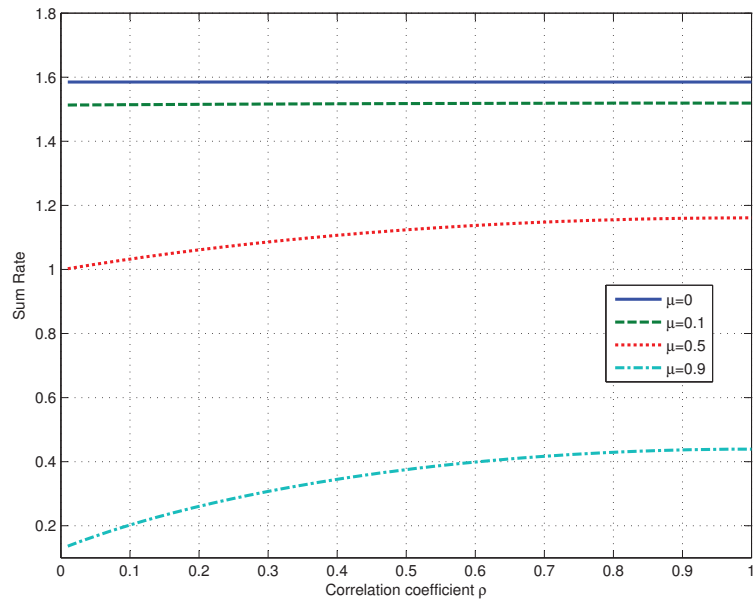


Fig. 8. Effect of side information's correlation on sum rate

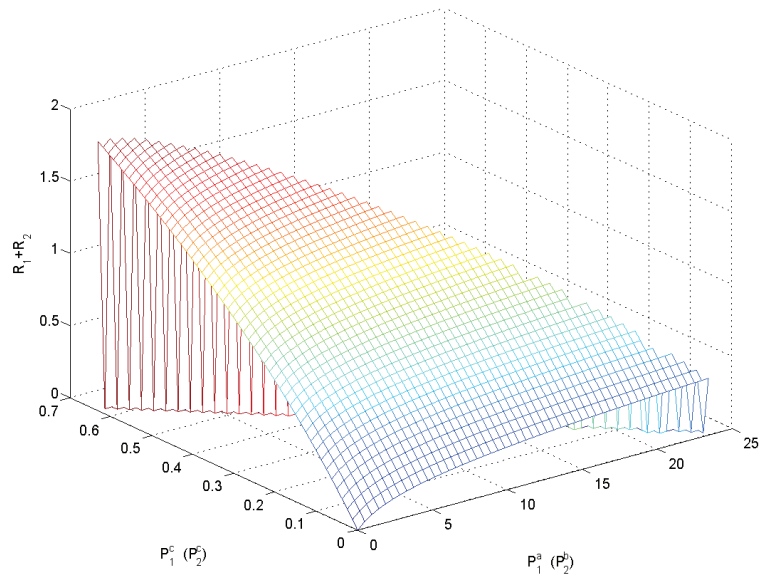


Fig. 9. Optimal power allocation on outer bounds 1,  $\mu = 0.1$ ,  $\rho = 0.5$

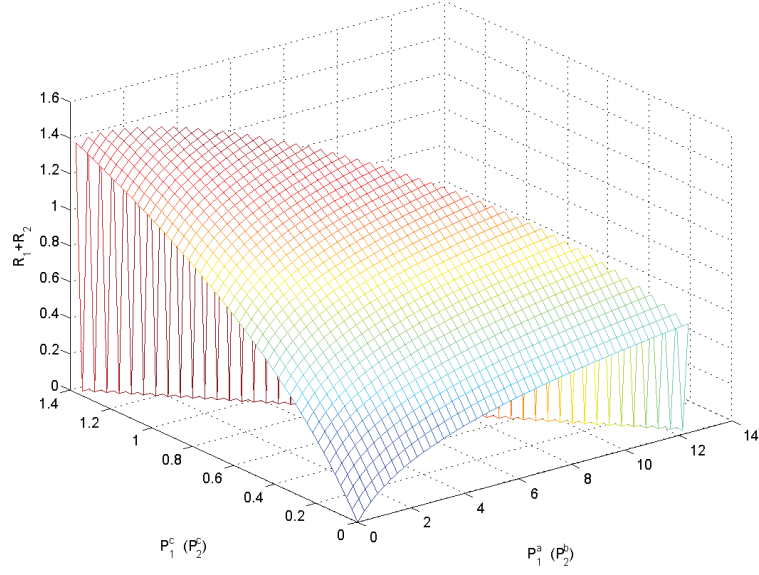


Fig. 10. Optimal power allocation on outer bounds 1,  $\mu = 0.1, \rho = 0$

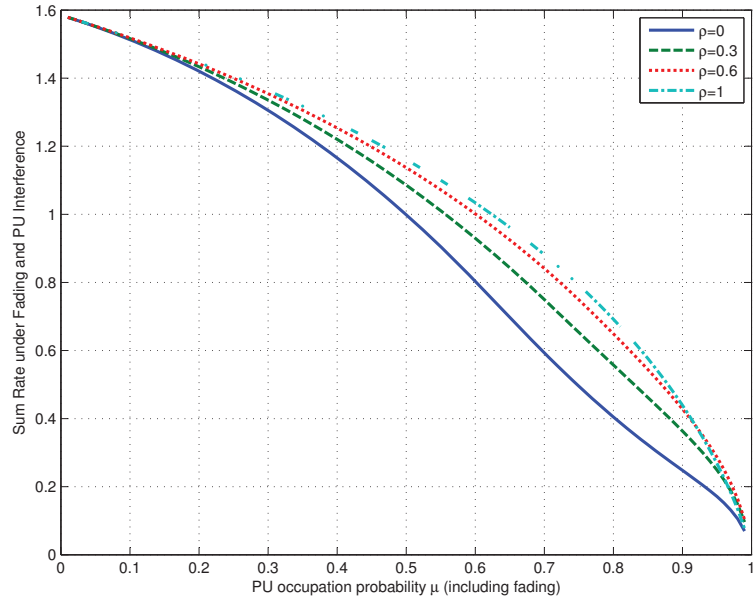


Fig. 11. Sum rate under fading and PU interference v.s.  $\mu$

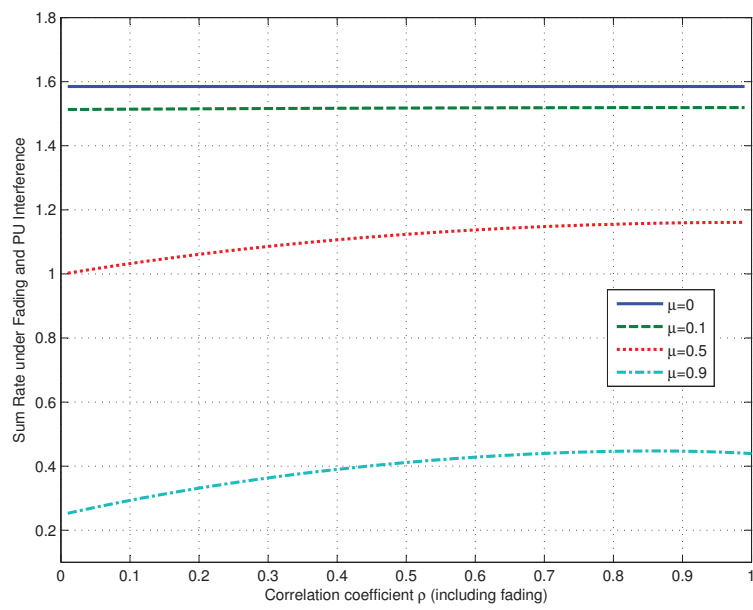


Fig. 12. Sum rate under fading and PU interference v.s.  $\rho$